Seat No.: _____ *Enrolment No._____*

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – WINTER • 2014

Subject code: 712901N **Date: 01-12-2014**

Subject Name: Mathematics for Researchers

Time: 10:30 am - 01:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Obtain Fourier series for the function $f(x) = x - x^2$ in the interval $[-\pi, \pi]$. Hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ... = \frac{\pi^2}{12}$. **07**

(b) Define Rounding off and explain rules to round off a number correct to n decimal places. **07**

Q.2 **(a)** Evaluate following integral by using Trapezoidal Rule **07**

$\int_4^{5.2} \log_e x \, dx$. (Take h=0.1)

(b) Detemine largest eigen value and corresponding eigen vectors of the following matrix correct to three decimal places by using power method. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **07**

OR

(b) Round-off the number 125250.5 up to four significant figures and compute Absolute, Relative and Percentage errors. **07**

Q.3 **(a)** A river is 80 ft wide. The depth y in a feet at a distance x ft. from one bank is given by the following table. **07**

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

Find the approximately the area the area of the cross section.

(b) Find $y'(1.1)$ and $y''(1.1)$ from the following table. **07**

x	1.1	1.2	1.3	1.4	1.5	1.6
y	8.403	8.781	9.129	9.451	9.750	10.031

OR

Q.3 **(a)** Express the function $f(x) = |x|$ as a Fourier series in the interval $[-\pi, \pi]$ **07**

(b) Find the Fourier transform of the function f defined as follows: **07**

$$f(x) = \begin{matrix} 1 - x^2; & |x| < 1 \\ 0 \quad ; & |x| > 1 \end{matrix}$$

Q.4 **(a)** Solve the following system of equations by using Gauss elimination method. **07**

$$10x - y + 2z = 4$$
$$x + 10y - z = 3$$
$$2x + 3y + 20z = 7$$

(b) State Cayley-Hamilton theorem and using it find the inverse of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ **07**

OR

Q.4 **(a)** By using Gauss-Jordan method find the inverse of the matrix $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ **07**

(b) Explain Escalator (Partition) method. **07**

Q.5 **(a)** Using Taylor's series method compute $y'(0.2)$ correct to five decimal places, given that $\frac{dy}{dx} = x^2 y - 1$, y(0)=1. **07**

(b) Apply Runge-Kutta method of order four to find an approximate value of y when x=0.2, given that $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, y(0)=1 **07**

OR

Q.5 **(a)** Find a real root of the equation $x^4 - x - 9 = 0$ correct to three decimal places by using Newton-Raphson method. **07**

(b) Find a real root of the equation $x^3 - 4x - 9 = 0$ correct to three decimal places by using Bisection Method. **07**