Seat No.: _____ *Enrolment No._____*

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – WINTER • 2014

Subject code: 714101N **Date: 01-12-2014**

Subject Name: Mathematical Methods in Signal Processing

Time: 10:30 am - 01:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all Questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q. 1)

a) Let $f : X \to R$ be an arbitrary function defined on a set X. Show that $d(x, y) = |f(x) - f(y)|$ is a Pseudometric. **[7]**

b) i) Explain why the set of real numbers is both open and closed. **[7]**

ii) Show that the boundary of a set S is a closed set.

Q. 2) **a)** Explain about stochastic AR and MA models. **[7]**

b) Explain about binary hypothesis testing. **[7]**

OR

b) Let (X,d) be a metric space show that $d_m(x, y) = \min(1, d(x, y))$ is a metric on X. **[7]**

Q. 3) **a)** What is the significance of HMM model? **[7]**

b) Obtain Z- transform of unit step and unit impulse. **[7]**

OR

Q. 3) **a)** **[7]**

A Grammian matrix R is always positive semi define. It is positive define if and only if the vectors $p_1, p_2, p_3, \ldots, p_m$, are linearly independent.

b) i) A linear operator $A : X \to Y$ is bounded if and only if it is continuousò. Justify the statement. **[7]**

ii) Compute transpose, inverse and rank of the matrix A ,

$$A = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{-2}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{10} \\ \frac{1}{5} & \frac{-4}{5} & \frac{1}{10} \end{bmatrix}$$

Q. 4) **a)** Explain Modulation Theorem and Parsevalôs Theorem. **[7]**

b) Explain sampling process with suitable example. Discuss about axtialiasing filter. **[7]**

OR

Q. 4) **a)** **[7]**

Show that if A has both a left inverse and a right inverse, they must be same.

b) **[7]**

Find eigen values and corresponding eigen vector of the matrix A

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Q. 5) **a)** **[7]**

Explain any two properties of z- transform.

b) **[7]**

Explain Linearity and scaling property of Fourier Transform.

OR

Q. 5) **a)** **[7]**

Let $A : H \to H$ be a bounded linear operator on a Hilbertspace H. Show that:

i) The adjoint operator A^* is linear.

ii) The adjoint operator A^* is bounded

iii) $\|A\| = \|A^*\|$.

b) **[7]**

Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and find A^{-1}

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