*Seat No.: _____* *Enrolment No._____*

# GUJARAT TECHNOLOGICAL UNIVERSITY

**M. E. - SEMESTER – I • EXAMINATION – WINTER • 2014**

**Subject code:** 714303 **Date: 03-12-2014**

**Subject Name:** Theory of Elasticity and Plasticity **Time: 10:30 am - 01:00 pm**

**Total Marks: 70**

## Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)** Derive basic differential equation in Cartesian co-ordinate system. **07**

**(b)** Derive strain-displacement relation in polar co-ordinate system. **07**

**Q.2** **(a)** Check the equilibrium of the following 2-D state of stresses without body forces **04**
ůx= 4x²+3xy-5y² ůy= x²+2xy+4y² Ůxy= -x²-8xy-(3/2)y²

**(b)** For the following state of stresses, find the principal stresses and the direction cosines of any **ONE** principal stress. Normal stresses: $\sigma_{xx}$=300 MPa, $\sigma_{yy}$ = 200 MPa, $\sigma_{zz}$ = 100 MPa, and Shear stresses: $\tau_{xy}$ = 0(zero) MPa, $\tau_{yz}$ = 0(zero) MPa, $\tau_{zx}$ = 0(zero) MPa, **10**

**OR**

**(b)** Using Swiftôs construction, find normal and resultant shear stress on a plane whose normal has direction cosines are l=0.848, m= 0.342 and n=0.405 respectively w.r.t principal stresses p1= 500 MPa(tensile), p2=150 MPa (tensile) and p3=400 MPa (compressive). **10**

**Q.3** **(a)** Discuss imperfection approach and state the principle of imperfection for stability of column. State the differential equation for the case of non-conservative forces for column with one end fixed & one end free using Dynamic criteria of stability. **07**

**(b)** For the curved beam subjected to moment: M = 150 kJ, internal & external radii: a = 150 mm & b = 350 mm respectively, calculate radial and transverse stresses at inner, outer and every quarter thickness points and plot their variations using the following equations with usual notations: **07**

**Radial stress:**

$\sigma_r = ï\ {}^{4M}/_{N}\ [a^2b^2/r^2 \ln (b/a) + b^2 \ln (r/b) + a^2 \ln (a/r)]$

**Tangential stress:**

$\sigma_\theta = ï\ {}^{4M}/_{N}\ [ï\ a^2b^2/r^2 \ln (b/a) + b^2 \ln (r/b) + a^2 \ln (a/r) + b^2 ï\ a^2]$

Here; $N = (b^2 ï\ a^2)^2 ï\ 4\ a^2b^2\ [\ln (b/a)]^2$

**OR**

**Q.3** **(a)** Derive the following equation with usual notations : **07**
$Ů_ď = Ů_x \cos^2 ď + Ů_y \sin^2 ď + Ů_{xy} \sin 2ď$

**(b)** Explain plane stress and plane strain problem. Also explain Generalized Hookôs law. **07**

**Q.4** **(a)** What is the concept of stability of structures? Give basis of stability of analysis for a slender straight column as well as column initially bent. **07**

**(b)** Derive equation of buckling load & deformation for the column with both end fixed which produces structural instability. **07**

**OR**

**Q.4** **(a)** Drawing neat sketch, explain the soap-bubble analogy of torsion in and derive the equation $\phi = (2\ C\ \theta\ S/p)\ z$ with usual notations. **07**

**(b)** Derive Airyôs stress function: $\phi = A \ln r + B\ r^2 \ln r + Cr^2 + D$ in Polar Co-ordinate System for an Axi-symmetric stress distribution. **07**

**Q.5** **(a)** Derive the equation of displacement for the column with one end hinged & other fixed in bent configuration which produces structural instability. **07**

**(b)** Find the linear strains: $\varepsilon_{xx}$, $\varepsilon_{yy}$ and shear strain: $\gamma_{xy,}$ if the linear strains measured by the strain gauges in the direction are $\varepsilon_{15^\circ} = 130x10^{ï 6}$ (Tensile), $\varepsilon_{65^\circ} = 665x10^{ï 6}$ (Compressive) and $\varepsilon_{125^\circ} = 271x10^{ï 6}$ (Tensile). Also, calculate the state of stresses. **07**

**OR**

**Q.5** **(a)** Derive the equation of displacement for the column, eccentrically loaded, with one end hinged & other fixed in bent configuration which produces structural instability. **07**

**(b)** Derive the basic equation of equilibrium for column in bent configuration subjected to dynamic force. Also explain mode shapes of buckling. **07**