Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER– III • EXAMINATION – WINTER • 2014

Subject code: 730403 **Date: 27-11-2014**

Subject Name:Optimization Techniques

Time: 02:30 pm - 05:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) (i) Find the maxima and minima of $f(x,y,z) = x^2 + y^2 + z^2$, given that $z^2 = xy + 1$ using direct substitution. **03**

(ii) Find the extreme points of the function **04**

$$f(x_1, x_2) = x_1^3 + x_2^3 - 63(x_1 + x_2) + 12x_1x_2.$$

(b) A wire of length $28\ m$ is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. Find the lengths of the two pieces so that the combined area of the square and the circle is minimum? **07**

Q.2 (a) Draw the flowchart of simplex method. **07**

(b) Use Simplex Method to maximize $f = 4x_1 + 6x_2 + 2x_3$ subject to **07**

$$x_1 + x_2 + x_3 \le 3, \quad x_1 + 4x_2 + 7x_3 \le 9, \qquad x_i \ge 0, \quad i = 1, 2, 3.$$

OR

(b) Use Revised Simplex Method to maximize $f = x_1 + 2x_2$ subject to $x_1 + x_2 \le 3,\ x_1 + 2x_2 \le 5,\ 3x_1 + x_2 \le 6,\ x_1 \ge 0,\ x_2 \ge 0.$ **07**

Q.3 (a) Use Dichotomous Search Method to find the minimum of $f = -x\sqrt{1-x}$ in the interval $(0.0, 1.00)$ to within 10% of the exact value. Take $\delta = 0.001$. **07**

(b) Use Newton's Method to minimize the function **07**

$$f(\lambda) = \lambda^3 - 6\lambda^2 + 9\lambda + 15$$

with $\lambda_0 = 1.5$. Iterate until $|f'(\lambda)| < 0.05$.

OR

Q.3 (a) Use Fibonacci Method with $n = 6$ to minimize the function **07**

$$f(\lambda) = \lambda^5 - 5\lambda^3 - 20\lambda + 5$$

in the interval $(0, 5)$.

(b) Use Secant Method to find the minimum of **07**

$$f(\lambda) = -\left(\frac{log\lambda}{\lambda}\right)$$

with an initial step size of $t_0 = 0.1, \lambda_1 = 1.5$, and $\varepsilon = 0.007$.

Q.4 (a) Use Interval Halving Method to minimize the function **07**

$$f(x) = x(x - 1.5)$$

in the interval $(0, 1)$. Perform three iterations.

(b) Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ using Powell's method with the starting point $(0, 0)$. Take $\varepsilon = 0.01$. **07**

OR

Q.4 (a) Use Golden Section Method to minimize the function **07**

$$f(\lambda) = \frac{0.5}{\sqrt{1+\lambda^2}} - \sqrt{1+\lambda^2}\left(1 - \frac{0.5}{1+\lambda^2}\right) + \lambda$$

with $n = 6$ in the interval $(0, 3)$.

(b) Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ using Univariate method with the starting point$(0, 0)$. Perform only one iteration. Take $\varepsilon = 0.01$. **07**

Q.5 (a) Use Cauchy's steepest descent method to minimize **07**

$$f(x_1, x_2) = 2x_1^2 + 2x_1x_2 + x_2^2 + x_1 - x_2$$

from the point $X_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$. Perform only three iterations.

(b) Find the dimensions of a rectangular prism type box that has the largest volume when the sum of its length, width and height is limited to a maximum value of 60 in. and its length is restricted to a maximum value of 36 in. **07**

OR

Q.5 (a) Minimize $f(x_1, x_2) = 6x_1^2 - 6x_1x_2 + 2x_2^2 - x_1 - 2x_2$ using Newton's Method from the starting point $X_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$. **07**

(b) Write the algorithm of Exterior Penalty function Method. Why do we have to normalize the constraints? **07**

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