

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
P.D.D.C. Sem- I Regular / Remedial Examination January. 2011

Subject code: X10001

Subject Name: Mathematics – I

Date: 03 / 01 /2011

Time: 10.30 am – 01.30 pm

Total Marks: 70

**Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** Do as directed. **02**
- (a) Which of the following matrices are in row-echelon form, reduced row-echelon form, both or neither. **03**
- $$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -7 & 5 & 5 \\ 0 & 1 & 3 & 2 \end{bmatrix}.$$
- (b) Using determinant method find the rank of a matrix **03**
- $$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}.$$
- (c) Using Gauss-Jordan method find the inverse of a matrix  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ . **03**
- (d) Evaluate :  $\int_0^5 \int_0^{x^2} x(x^2 + y^2) dx dy$ . **03**
- (e) Find  $\text{curl } \vec{F}$ , where  $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$ . **03**
- Q.2** (a) 1) Solve **04**
- $$\begin{aligned} -2y + 3z &= 1 \\ 3x + 6y - 3z &= -2 \\ 6x + 6y + 3z &= 5 \end{aligned}$$
- By Gaussian elimination and back-substitution.
- 2) Solve :  $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$ . **03**
- (b) 1) If  $u = \tan^{-1}\left(\frac{y^2}{x}\right)$ , prove that **04**
- $$x \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\sin^2 u \sin 2u.$$
- 2) If  $u = \frac{yz}{x}$ ,  $v = \frac{zx}{y}$ ,  $w = \frac{xy}{z}$ , Find  $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ . **03**
- OR**
- (b) 1) If  $z(x+y) = x^2 + y^2$ , then show that  $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$ . **04**
- 2) Solve :  $x \frac{dy}{dx} = y^2 + y$ . **03**

- Q.3 (a)** Find the eigen values and eigen vectors of a matrix  $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ . **05**
- (b)** Find the extreme values of a function  $f(x, y) = x^3 y^3 (1 - x - y)$ . **05**
- (c)** Find the orthogonal trajectories of the family of parabola  $y = ax^2$ . **04**

**OR**

- Q.3 (a)** Solve :  $x \log x \frac{dy}{dx} + y = \log x^2$ . **05**
- (b)** Evaluate  $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$  by changing to polar coordinates. **05**
- (c)** Solve :  $(x^2 + y^2 - a^2)xdx + (x^2 - y^2 - b^2)ydy = 0$ . **04**
- Q.4 (a)** Trace the curve  $y^2(a - x) = x^2(a + x)$ . **05**
- (b)** Change the order of integration in  $I = \int_0^1 \int_{x^2}^{2-x} xy dx dy$ , and hence evaluate the same. **05**
- (c)** Evaluate :  $\int_0^4 \int_0^x \int_0^{x+y} z dz dy dx$ . **04**

**OR**

- Q.4 (a)** Trace the curve  $r^2 = a^2 \cos 2\theta$ . **05**
- (b)** Find, by double integration, the area lying between the parabola  $y = 4x - x^2$  and the line  $y = x$ . **05**
- (c)** Solve :  $\frac{dy}{dx} + y \tan x = y^2 \sec x$ . **04**
- Q.5 (a)** Find the directional derivative of  $\phi = x^2 yz + 4xz^2$  at the point  $(1, -2, -1)$  in the direction of the vector  $2\bar{i} - \bar{j} - 2\bar{k}$ . **05**
- (b)** If  $\vec{F} = (5xy - 6x^2)\bar{i} + (2y - 4x)\bar{j}$ , evaluate  $\int_C \vec{F} \cdot d\vec{r}$  along the curve C in the  $xy$ -plane,  $y = x^3$  from the point  $(1,1)$  to  $(2,8)$ . **05**
- (c)** If  $u = x^2 + y^2 + z^2$ , and  $\vec{r} = x\bar{i} + y\bar{j} + z\bar{k}$ , then find  $\text{div}(u\vec{r})$  in terms of  $u$ . **04**

**OR**

- Q.5 (a)** Verify Green's theorem for  $\int_C [(3x - 8y^2)dx + (4y - 6xy)dy]$  where C is the boundary of the region bounded by  $x = 0$ ,  $y = 0$  and  $x + y = 1$ . **05**
- (b)** Find constant 'a' so that  $\vec{F} = y(ax^2 + z)\bar{i} + x(y^2 - z^2)\bar{j} + 2xy(z - xy)\bar{k}$  is solenoidal. **05**
- (c)** Prove that  $\nabla r^n = nr^{n-2}\vec{r}$ , where  $\vec{r} = x\bar{i} + y\bar{j} + z\bar{k}$ . **04**

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