

**GUJARAT TECHNOLOGICAL UNIVERSITY**

P.D.D.C. Sem- II Remedial Examination Nov / Dec. 2010

Subject code: X20001

Subject Name: Mathematics-2

Date: 27 / 11 / 2010

Time: 10.30 am – 01.30 pm

**Instructions:****Total Marks: 70**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

**Q.1** Do as directed. **14**

(a) Define Beta function. Prove that

$$\beta(m, n) = 2 \int_0^{\frac{\pi}{2}} \sin^{2m-1} \theta \cos^{2n-1} \theta d\theta$$

(b) Prove that  $\beta(m, n+1) = \frac{n}{m+n} \beta(m, n)$ .(c) Define Gamma Function. Prove that  $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$ .(d) Prove that  $\int_0^{\frac{\pi}{2}} \sqrt{\sin \theta} d\theta \times \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin \theta}} = \pi$ .(e) Show that  $\int_0^{\infty} x^2 e^{-x^4} dx = \frac{1}{4} \sqrt{\frac{3}{4}}$ .(f) Prove that  $\int_0^1 \frac{dx}{\sqrt{-\log x}} = \sqrt{\pi}$ .(g) Prove that  $\int_0^1 \frac{1}{2} = \sqrt{\pi}$ .**Q.2** (a) Find the Laplace transform of the following: ( any two ) **04**(i)  $e^{3t} \sin^2 t$  (ii)  $t \cos 2t$  (iii)  $\frac{\cos at - \cos bt}{t}$ (b) State convolution theorem. Using it find inverse Laplace transform of  $\frac{1}{(s+2)(s+3)}$ . **03**(c) (I) Find the Laplace transform of the following: ( any two ) **03**(i)  $\cos 2t \sin 2t$  (ii)  $t e^{-t} \cos 2t$  (iii)  $\frac{1-e^t}{t}$ (II) Find the inverse Laplace transforms of **04**(i)  $\frac{s+2}{s^2-4s+13}$  (ii)  $\log\left(\frac{s+1}{s-1}\right)$ .**OR**(c) (I) Find the inverse Laplace transforms of **04**(i)  $\tan^{-1}\left(\frac{2}{s^2}\right)$  (ii)  $\frac{2s^2-6s+5}{(s-1)(s-2)(s-3)}$ .(II) By using method of Laplace transform solve the initial value problem  $y'' + 4y' + 3y = e^{-t}$ ,  $y(0) = y'(0) = 1$ . **03****Q.3** (a) Solve: **05**(i)  $(D^2 + 5D + 6)y = e^x$

(ii)  $(D^2 - 5D + 6)y = \sin 3x$ .

- (b) Using the method of variation of parameter solve the differential equation  $y'' + y = \sec x$ . 05
- (c) Solve  $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = \log x$ . 04

**OR**

- Q.3** (a) Solve: 05

(i)  $(D^2 + D)y = x^2 + 2x + 4$

(ii)  $(D^2 - 2D + 4)y = e^x \cos x$ .

- (b) Solve  $(1+x)^2 \frac{d^2 y}{dx^2} + (1+x) \frac{dy}{dx} + y = 2 \sin(\log(1+x))$ . 05

- (c) Show that the frequency of free vibration in a closed electrical Circuit with inductance L and capacity C in series is  $\frac{30}{\pi\sqrt{LC}}$  per minute. 04

- Q.4** (a) Find the Fourier series of the function  $f(x) = x^2, -\pi < x < \pi$ . 05

- (b) Find the Fourier series expansion for  $f(x)$ , if  $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$  05

- (c) Find the Fourier transform of  $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$  04

Hence evaluate  $\int_0^{\infty} \frac{\sin x}{x} dx$

**OR**

- Q.4** (a) Obtain the Fourier series for  $f(x) = e^{-x}$  in the interval  $0 < x < 2\pi$ . 05

- (b) Express  $f(x) = x$  as a half range sine series in  $0 < x < 2$ . 05

- (c) Find the Fourier sine transform of  $\frac{e^{-ax}}{x}$ . 04

- Q.5** (a) Form the partial differential equation from: 05

(i)  $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

(ii)  $z = f(x^2 - y^2)$ .

- (b) Solve: 05

(i)  $(mz - ny) \frac{\partial z}{\partial x} + (nx - lz) \frac{\partial z}{\partial y} = ly - mx$

(ii)  $((x^2 - y^2 - z^2)p + 2xyq = 2xz)$ .

- (c) (i) State and prove linearity property of Z transform. 04  
(ii) state and prove Damping rule of Z transform.

**OR**

- Q.5** (a) Solve by the method of separation of variable 05

$$\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

- (b) Solve (i)  $\sqrt{p} + \sqrt{q} = 1$  05

(ii)  $p^2 + q^2 = x + y$ .

- (c) Define Z transform. 04

Solve (i)  $z(a^n)$

(ii)  $z(n^p)$ .

\*\*\*\*\*