Seat No.: _________ Enrolment No.______________

GUJARAT TECHNOLOGICAL UNIVERSITY
PDDC SEM-I Examination-Dec-2011

Subject code: X10001 **Date: 19/12/2011**
Subject Name: Mathematics-I
Time: 10.30 am -1.30 pm **Total marks: 70**

Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) i. Determine the rank of $A=\begin{bmatrix}1&2&3\\1&4&2\\2&6&5\end{bmatrix}$. **02**

ii. If $x = r\cos\theta,\ y = r\sin\theta$ show that $\frac{\partial r}{\partial x} = \frac{\partial x}{\partial r}$. **02**

iii. What is degree and order of $\frac{dy}{dx} = 2xy$? Solve it. **03**

(b) i. Discuss the nature of the origin for $y^2 = x(x+2) - 3$. **01**

ii. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} dydx$ by changing into polar coordinates. **03**

iii. Find a vector normal to the surface $xy^3z^2 = 4$ at the point $(-1, -1, 2)$. **03**

Q.2 (a) i. Solve : $x + 2y + 3z = 0, 3x + 4y + 4z = 0, 7x + 10y + +12z = 0$. **03**

ii. Using Gauss-Jordan method find the inverse of $B = \begin{bmatrix}3&-3&4\\2&-3&4\\0&-1&1\end{bmatrix}$. **04**

(b) i. Find the eigen values of $C = \begin{bmatrix}2&0&1\\0&2&0\\1&0&2\end{bmatrix}$. Find eigen vector corresponding to its smallest eigen value. **04**

ii. If $u = e^{xyz}$, find u_{xyz}. **03**

OR

(b) i. If $D = \begin{bmatrix}1&5\\0&-1\end{bmatrix}$ find the eigen values of D^9. **03**

ii. Find u_{xy} for $u = x^2\tan^{-1}\frac{y}{x} - y^2\tan^{-1}\frac{x}{y}$. **04**

Q.3 (a) i. If $u = \sin^{-1}\frac{x^2y^2}{x+y}$, show that $xu_x + yu_y = 3\tan u$. **03**

ii. If $x = r\cos\theta,\ y = r\sin\theta$, prove that $JJ' = 1$. **04**

(b) Solve : **07**

i. $xy\frac{dy}{dx} = 1 + x + y + xy$

ii. $\left(x^2-y^2\right)dx - xydy = 0$

OR

Q.3 (a) i. Examine $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for exreme values. **04**

ii. In estimating the cost of a pile of bricks measured as $2m \times 15m \times 1.2m$, the tape is stretched 1% beyond the standard length. Find the percentage error in the volume of the pile. **03**

(b) Solve : **07**

i. $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$

ii. $\left(x^2 - ay\right) dx = \left(ax - y^2\right) dy$

Q.4 **(a)** If the stream lines of a flow around a corner are $xy =$ constant, find their orthogonal trajectories. **04**

(b) Trace the curves **10**

i. $r^2 = a^2 \cos 2\theta$

ii. $x^2 y = a^2 (a - y)$; $a > 0$

OR

Q.4 **(a)** When a resistance R ohms is connected in series with an inductance L henries with an e.m.f. of E volts, the current i amperes at time t is given by $L\frac{di}{dt} + Ri = E$. If $E = 10 \sin t$ volts and $i = 0$ when $t = 0$, find i as a function of t. **04**

(b) Trace the curves: **10**

i. $y^2(2a - x) = x^3$; $a > 0$

ii. $r = a(1 + \cos\theta)$; $a > 0$

Q.5 **(a)** i. Evaluate by changing the order of integration $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dy dx$. **03**

ii. By triple integration, find the volume of the sphere $x^2 + y^2 + z^2 = 1$. **04**

(b) i. Find the value of a if $\left(ax^2y + yz\right) i + \left(xy^2 - xz^2\right) j + \left(2xyz - 2x^2y^2\right) k$ is solenoidal. **03**

ii. Varify Green's theorem for $\int_C \left[\left(xy + y^2\right)dx + x^2 dy\right]$, where C is bounded by the curves $y = x$, $y = x^2$. **04**

OR

Q.5 **(a)** i. Evaluate $\int_{-1}^{1} \int_0^z \int_{x-z}^{x+z} (x + y + z)\, dy dx$ **03**

ii. Find, by double integration, the area lying inside the curve $r = 2a \cos\theta$ **04**

(b) i. Find the curl of $\left(-2x^2y + yz\right) i + \left(xy^2 - xz^2\right) j + \left(2xyz - 2x^2y^2\right) k$. **03**

ii. Apply Green's theorem to evaluate $\int_C \left[\left(2x^2 - y^2\right)dx + \left(x^2 + y^2\right)dy\right]$, where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = 1$. **04**

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