

GUJARAT TECHNOLOGICAL UNIVERSITY**PDDC - SEMESTER-I - EXAMINATION – SUMMER 2017****Subject Code: X10001****Date: 26/05/2017****Subject Name: MATHEMATICS-1****Time: 02:30 PM to 05:30 PM****Total Marks: 70****Instructions:**

1. Attempt any five questions.
1. Make suitable assumptions wherever necessary.
2. Figures to the right indicate full marks.

- Q.1 (a)** Find the inverse of the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ by Gauss-Jordan method **07**
- (b)** Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ by using row operation method and determinant method also. **07**
- Q.2 (a)** Trace the cissoids $y^2(2a - x) = x^3$. **07**
- (b)** Find the Eigen value and Eigen vector of the matrix $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$. **07**
- Q.3 (a)** If $u = f(x - y, y - z, z - x)$ the show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **07**
- (b)** If $u = \frac{1}{3} \log \left(\frac{x^3 + y^3}{x^3 - y^3} \right)$, find the value of **07**
- (i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ (ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$.
- Q.4 (a)** Find the extreme value of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. **07**
- (b)** Prove that $JJ^* = 1$, where $x = r \cos \theta$, $y = r \sin \theta$ and J is Jacobian determinant. **07**
- Q.5 (a)** Find the directional derivative of $\phi = 6x^2y + 24y^2z - 8z^2x$ at (1,1,1) in the direction parallel to the line $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$. Hence, find its maximum value. **07**
- (b)** Verify Green's theorem for $\int_C [(x^2 - 2xy)dx + (x^2y + 3)dy]$, where C is the boundary of the region bounded by the parabola $y = x^2$ and the line $y = x$. **07**
- Q.6 (a)** Change the order of integration and evaluate $\int_0^{\infty} \int_0^x x e^{-y} dy dx$. **07**
- (b)** Solve $\frac{dy}{dx} + \frac{y}{x} = y^3$. **07**

Q.7 (a) Evaluate (i) $\int_0^1 \int_0^2 (x^2 + y^2) dy dx$, (ii) $\int_0^{\log 2} \int_0^x \int_0^{x+y} e^{(x+y+z)} dz dy dx$

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(b) Solve $\frac{dy}{dx} = \cos x \cos y - \sin x \sin y$.

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