

**GUJARAT TECHNOLOGICAL UNIVERSITY****PDDC- SEMESTER-II - EXAMINATION – SUMMER 2017017****Subject Code: X20001****Date:26/05/2017****Subject Name: MATHEMATICS-2****Time: 10:30 AM to 01:30 PM****Total Marks: 70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) Use the method of variation of parameter  $y'' - 4y' + 4y = \frac{e^{2x}}{x^2}$ . **07**
- (b) (I) Prove that  $L(t^n) = \frac{n!}{s^{n+1}}, n > 0$ . (II)  $L(e^{-at}) = \frac{1}{s+a}$ . **07**
- Q.2** (a) Find the Fourier series to represent  $e^{ax}$  in the interval  $-\pi$  to  $\pi$ . **07**
- (b) By the Laplace transform, solve  $y'' + a^2y = k \sin at, y(0) = \alpha$  &  $y'(0) = \beta$ . **07**
- Q.3** (a) Find the inverse Laplace transform of  $\frac{5s+3}{(s^2+2s+5)(s-1)}$ . **07**
- (b) Find the Fourier cosine series for  $f(x) = x^2, 0 < x < c$ . **07**
- Q.4** (a) Using the method of separation of variables to solve  $u_x = 2u_t + u$ , given that  $u(x, 0) = e^{-3x}$ . **07**
- (b) Find the Z- transform of (I)  $a^k$ , (II)  $c^k \sin \alpha k, k \geq 0$ . **07**
- Q.5** (a) (I) Find the complete integral of  $q = pq + p^2$ . **07**
- (II) Find the PDE of the of all spheres whose centers lie on the z-axis. **07**
- (b) (I) Solve  $y''' - y'' + 100y' - 100y = 0, y(0) = 4, y'(0) = 11$  &  $y''(0) = -299$  **07**
- (II) Solve  $y'' - 4y = 0$ . **07**
- Q.6** (a) (I) Prove that  $B(m, n) = B(n, m)$ , (II)  $\Gamma_{n+1} = n!$ , where  $\Gamma$  is Gamma function. **07**
- (b) Find the general solution of  $y''' - 3y'' + 3y' - y = 4e^t$ . **07**
- Q.7** (a) Using Fourier integral show that  $\int_0^\infty \frac{1 - \cos \pi \lambda}{\lambda} \sin(x\lambda) d\lambda = \begin{cases} \frac{\pi}{2}, 0 < x < \pi \\ 0, x > \pi \end{cases}$  **07**
- (b) Prove that  $\left\{ \left( \int_0^\infty \sqrt{x} e^{-x^2} dx \right) \cdot \left( \int_0^\infty \frac{e^{-x^2}}{\sqrt{x}} dx \right) \right\} = \frac{\pi}{2\sqrt{2}}$ . **07**

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