

Seat No.: \_\_\_\_\_

Enrolment No. \_\_\_\_\_

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
**PDDC - SEMESTER-II • EXAMINATION – SUMMER 2013**

**Subject Code: X20001**

**Date: 04-06-2013**

**Subject Name: Mathematics-II**

**Time: 02.30 pm - 05.00 pm**

**Total Marks: 70**

**Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1**
- (a) (i) Define Gamma and Beta Functions. **02**
- (ii) Show that  $\int_0^{\infty} \frac{e^{-3t}}{\sqrt{t}} dt = \sqrt{\frac{\pi}{3}}$ . **03**
- (iii) Find  $L(t \sin t)$ . **02**
- (b) (i) Find  $L(f(t))$  if  $f(t) = \begin{cases} 1, 0 < t < 2 \\ 3, t > 2. \end{cases}$ . **03**
- (ii) Find the inverse Laplace transform of  $\frac{3s-2}{(s+2)(s^2+1)}$ . **03**
- (iii) State Relation between Gamma and Beta Functions. **01**
- Q.2**
- (a) (i) State Convolution theorem. Using it find  $L^{-1}\left(\frac{1}{(s+1)(s+2)}\right)$  **04**
- (ii) Solve :  $y'' + y = t$ ,  $y(0) = 0$  &  $y'(0) = 1$ . Using Laplace transform. **03**
- (b) (i) Evaluate  $\int_0^{\infty} e^{-t} \cosh t dt$  by using Laplace transform. **03**
- (ii) Show that  $\int_0^{\frac{\pi}{2}} \sqrt{\cot \theta} d\theta = \frac{1}{2} \left( \frac{1}{4} \right) \frac{3}{4}$ . **04**
- OR**
- (b) (i) Find the inverse Laplace transform of  $\log\left(\frac{s+2}{s+5}\right)$ . **03**
- (ii) Evaluate  $\int_0^1 \frac{dx}{(1-x^3)^{1/3}}$  by using Gamma-Beta functions. **04**
- Q.3**
- (a) Find the Fourier series expansion of  $f(x) = x - x^2, -\pi < x < \pi$ . **05**
- (b) Find the Fourier series expansion of  $f(x) = x^3, -2 < x < 2$ . **05**
- (c) Find the Fourier sine transform of  $f(x) = e^{-2x}$ . **04**
- OR**
- Q.3** (a) Find the Fourier series expansion of  $f(x) = e^{-x}, 0 < x < 2\pi$ . **05**

(b) Find the Fourier cosine transform of  $f(x) = \begin{cases} x^2, & -1 < x < 0 \\ 1+x, & 0 < x < 1 \\ 0, & x > 1. \end{cases}$  05

(c) Find the Fourier sine series of  $f(x) = 3 - x, 0 < x < 3$ . 04

**Q.4** (a) Solve  $(D^2 + 3D + 2)y = x^2 + e^{-x}$ . 05

(b) Using the method of variation of parameter, solve  $y'' + y = \cos ecx$ . 05

(c) Solve  $y'' + 9y = 3x^2$ . 04

**OR**

**Q.4** (a) Solve  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$ . 05

(b) Solve  $x^2 y'' - xy' + 4y = \cos(\log x) + x \sin(\log x)$ . 05

(c) Solve the simultaneous equations:  $\frac{dx}{dt} = -wy, \frac{dy}{dt} = wx$ . 04

**Q.5** (a) Form the partial differential equation from 05

(i)  $z = f(x^2 + y^2)$ , (ii)  $f(xy + z^2, x + y + z) = 0$ .

(b) Using the method of separation of variables, solve  $u_{xx} = 25u_y$ . 05

(c) Define Z-transform. Find the Z-transform of the sequence  $\{a^m\}, m \geq 0$ . 04

**OR**

**Q.5** (a) Solve : 05

(i)  $\frac{\partial^2 z}{\partial x^2} = \cos x$ , (ii)  $\frac{\partial^2 z}{\partial x \partial y} = x^2 + y^2$

(b) Solve  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  by using the method of separation of variables. 05

(c) State the linearity property of Z-transform. Find the Z-transform of 04

$\{f(k)\}$ , where  $f(k) = \begin{cases} 7^k, & k < 0 \\ 5^k, & k \geq 0. \end{cases}$

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