

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
**PDDC - SEMESTER-II • EXAMINATION – SUMMER • 2014**

**Subject Code: X20001****Date: 18-06-2014****Subject Name: Mathematics-II****Time: 10:30 am – 01:30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

**Q.1 (a)**      1) Prove that  $\beta(m, n) = \beta(m+1, n) + \beta(m, n+1)$ . 03

2) Define Gamma function and evaluate  $\int_0^{\frac{\pi}{2}} \sqrt{\cot \theta} d\theta$ . 04

**(b)** Using the Fourier integral represent, show that  $\int_0^{\infty} \frac{\cos \omega x}{1+\omega^2} d\omega = \frac{\pi}{2} e^{-x}$  ( $x \geq 0$ ). 07

**Q.2 (a)** Find the Fourier series of the function  $f(x) = x^2$ ,  $-\pi < x < \pi$ ,  
 $f(x+2\pi) = f(x)$ . 07

**(b)** Solve  $(D^2 - 4D + 3)y = \sin 3x \cos 2x$ . 07

**OR**

**(b)** Solve  $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = e^x \cos x$ . 07

**Q.3 (a)** Find the Fourier series expansion for  $f(x)$  if 07

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}, \quad f(x+2\pi) = f(x).$$

**(b)** Solve by the method of variation of parameter  $\frac{d^2y}{dx^2} + y = \sec x$ . 07

**OR**

**Q.3 (a)** Find the half-range sine series of the function  $f(x) = e^x$  in  $0 < x < 1$ ,  
 $f(x+2) = f(x)$ . 07

**(b)** Solve  $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^4$ . 07

**Q.4 (a)**      1) Find the Laplace transform of  $f(t) = e^{2t} + 4t^3 - 2 \sin 3t + 3 \cos 3t$  03

2) Find the inverse Laplace transform of  $\tan^{-1}\left(\frac{2}{s}\right)$ . 04

**(b)** Solve  $(y+z)p - (z+x)q = x - y$ . 07

**OR**

**Q.4 (a)**      1) Find the Laplace transform of  $f(t) = t \cos at$ . 03

2) Find the inverse Laplace transform of  $\frac{3s+2}{s^2 - s - 2}$ . 04

**(b)** Using the method of separation of variables solve  $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$ ; given that  
 $u(0, y) = 8e^{-3y}$ . 07

**Q.5** (a) 1) Form the partial differential equation from  $(x-a)^2 + (y-b)^2 + z^2 = c^2$ . **03**  
2) Solve  $z = p^2 + q^2$ . **04**

(b) Use Laplace method to solve  $y'' + y' = t$ ;  $y(0) = 1$ ,  $y'(0) = 0$ . **07**

**OR**

**Q.5** (a) 1) Form the partial differential equation from  $z = f(x^2 + y^2) + x + y$ . **03**  
2) Solve  $p^2 + q^2 = x^2 + y^2$ . **04**

(b) Define the convolution theorem and using it evaluate  $L^{-1}\left\{\frac{s}{(s+2)(s^2+9)}\right\}$ . **07**

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