

GUJARAT TECHNOLOGICAL UNIVERSITY
PDDC - SEMESTER- I EXAMINATION – SUMMER 2015

Subject Code: X10001**Date:04/06/2015****Subject Name: MATHEMATICS-1****Time:02.30pm-05.30pm****Total Marks: 70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) Find the eigen values and corresponding eigen vectors of the following matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

07

(b) (i) Find the rank of matrix $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{bmatrix}$ by determinant method.

03

(ii) If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x+y}}\right)$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0$

04

Q.2 (a) Trace the curve $y^2(a+x) = x^2(a-x)$

07

(b) (i) If $u = (x^2 + y^2 + z^2)^{-1/2}$; then prove that

$$(I) \quad x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = -u \quad (II) \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

07

Q.3 (a) Evaluate $\iint_A y dA$, where A is the region of integration bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

07

(b) (i) Evaluate $\int_0^2 \int_1^z \int_0^{xz} xyz \, dx \, dy \, dz$

04

(ii) Solve $(xy^2 + 2x^2y^3) dx + (x^2y - x^3y^2) dy = 0$

03

Q.4 (a) Find the maximum and minimum value of the function $f(x, y) = x^3y^2(1-x-y)$

07

(b) (i) Solve the following system of linear equations by Gauss-Elimination method:
 $x + y + z = 6$, $x + 2y + 3z = 14$, $x + 4y + 9z = 36$

04

(ii) Find the directional derivative of $f = x^2 - 2y^2 + 4z^2$ at the point $(1, 1, -1)$ in the direction of the vector $2\hat{i} + \hat{j} + \hat{k}$.

03**Q.5**

(a) Find the Inverse of a matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$

07

(b) (i) If $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$, $z = r \cos \theta$ then find $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$

03

(ii) Find the orthogonal trajectories of the family of rectangular hyperbolas $x^2 - y^2 = a^2$ 04

Q.6 (a) Change the order of integration and evaluate $\int_0^x \int_0^{\infty} x e^{-x^2/y} dA$. 07

(b) (i) Evaluate $\int_0^{\pi/4} \int_0^{\sqrt{\cos 2\theta}} \frac{r}{1+r^2} dr d\theta$ 03

(ii) Solve $\frac{dy}{dx} = e^{(x-y)} (e^x - e^y)$ 04

Q.7 (a) Verify the Green theorem for $\oint (xy + y^2) dx + x^2 dy$, where C is boundary of the region bounded by $y = x$ and $y = x^2$ 07

(b) (i) If $f = (ax + 3y + 4z)\hat{i} + (x - 2y + 3z)\hat{j} + (3x + 2y - z)\hat{k}$ is a solenoidal vector, find 'a'. 04

(ii) Solve $(e^y + 1) \cos x dx + e^y \sin x dy = 0$ 03
