

GUJARAT TECHNOLOGICAL UNIVERSITY
PDDC - SEMESTER- II EXAMINATION – SUMMER 2015

Subject Code: X20001**Date: 29/05/2015****Subject Name: Mathematics - II****Time: 10.30am-01.30pm****Total Marks: 70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1

(a) 1) Define Beta Function. Show that $B(m, n) = 2 \int_0^{\pi/2} \sin^{2m-1} \theta \cdot \cos^{2n-1} \theta d\theta$ **04**

2) Evaluate $\int_0^{\infty} \frac{x^4}{4^x} dx$ **03**

(b) Solve the differential equation by Laplace transform. **07**
 $y'' - y' - 6y = 0, \quad y(0) = 6, \quad y'(0) = 13$

Q.2 (a) 1) Find Laplace transform of $f(t) = t + \sin t + 2t^2 + 3 \cos 3t + e^t$ **03**

2) Find Laplace inverse of $F(s) = \frac{1}{(s-1)(s-3)(s-5)}$ **04**

(b) State convolution theorem. Using convolution theorem find $L^{-1}\left(\frac{s}{(s^2+1)^2}\right)$ **07**

Q.3 (a) Find Fourier series of $f(x) = x + x^2, \quad x \in (0, 2\pi), \quad f(x+2\pi) = f(x)$ **07**

(b) Find Fourier series of 2π -periodic function $f(x) = \begin{cases} x & \text{if } -\pi < x < 0 \\ \pi - x & \text{if } 0 < x < \pi \end{cases}$ **07**

Q.4 (a) Find half-range Fourier Cosine series of $f(x) = x^2, \quad 0 < x < 1, \quad f(x+2) = f(x)$ **07**

(b) Solve : $y'' + 2y' + y = 2x \sin x$ **07**

Q.5 (a) Solve by variation of parameters method : $y'' + y = \cos ecx$ **07**

(b) Solve : $(x^2 D^2 + 6xD + 6)y = x^2$ **07**

Q.6 (a) 1) Form partial differential equation of $z = (x-1)^2 + (y-2)^2$ **03**

2) Solve : $x(y-z)p + y(z-x)q = z(x-y)$ **04**

(b) Using the method of separation of variables, solve the partial differential equation **07**
 $u_{xx} = 16u_y$

Q.7 (a) Find Fourier transform of the function $f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ **07**

(b) 1) Define z-transform. Find z- transform of $f(k) = e^{-akT}, k \geq 0$ **03**

2) Find the inverse z-transform of the function $F(z) = \frac{z+1}{z^2 + 0.3z + 0.02}$ **04**