Seat No.: _____ *Enrolment No.______*

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC SEM-II Examination May 2012

Subject code: X20001

Subject Name: Mathematics-II

Date: 22/05/2012 **Time: 10.30 am – 01.30 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) (i) $\beta(3,4) =$ ___________ **01**

(ii) Write relation between Beta and Gamma function. **01**

(iii) Show that $\beta(m,n) = \beta(n,m)$ **01**

(iv) Define Gamma function **01**

(v) Express integral $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$ in term of gamma function. **03**

(b) (i) Find the period of cos2x **02**

(ii) Find $L(e^{4t} + \cos 3t + t^4)$ **02**

(iii) Solve differential equation $D^2y - a^2y = 0$ **03**

Q.2 (a) (i) Find $L^{-1}\left(\frac{2s^2-6s+5}{(s-1)(s-2)(s-3)}\right)$ **04**

(ii) Find $L(e^{-3t}(\sin 5t - \cos 5t))$ **03**

(b) (i) Use Convolution theorem to evaluate $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$ **03**

(ii) Using Laplace transform solve $y'' - y = e^{2t}$, $y(0) = y'(0) = 0$ **04**

OR

(b) (i) Evaluate $L\left(e^{3t}\int_0^t \frac{\sin t}{t}dt\right)$ **03**

(ii) Using Lapalce transform solve $y'' - 2y' + y = e^{t}$, $y(0) = 2$, $y'(0) = -1$. **04**

Q.3 (a) (i) Find the Fourier series for $f(x) = x^3, -\pi \le x \le \pi, f(x+2\pi) = f(x)$. **03**

(ii) Find the Fourier series for $f(x) = 1$ if $0 \le x \le \pi$ and $f(x) = 0$ if $\pi \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$. **04**

(b) Find a Fourier series for $f(x) = x - x^2$ from $x = -\pi$ to $x = \pi$. Hence prove that **07**

$$\frac{\pi^2}{12} = \frac{1}{1^1} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$

OR

Q.3 (a) (i) Express $f(x) = x$ as a half range cosine series in $0 < x < 2$. **03**

(ii) Express $f(x) = 1$ for $0 \le x \le \pi$ and $f(x) = 0$ for $x > \pi$ as Fourier sine integral and hence evaluate $\int_0^\infty \frac{1-\cos \pi\lambda}{\lambda} \sin x\lambda d\lambda$ **04**

(b) (i) Expand $\pi x - x^2$ in half range sine series in the interval $(0,\pi)$ up to the first three term. **03**

Q.3 **(b)** (ii) Find the Fourier sine transform of $e^{-|x|}$. Hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2} dx$. **04**

Q.4 **(a)** In an L-C-R circuit, the charge q on a plate of a condenser is given by $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E \sin pt$. The circuit is tuned to resonance so that $p^2 = 1/LC$. If initially the current i and the charge q be zero, show that, for small values of R/L, the current in the circuit at time t is given by (Et/2L) sin(pt). **07**

(b) (i) Solve differential equation $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2$ **03**

(ii) Solve differential equation **04**

$(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin(\log(1+x))$

OR

Q.4 **(a)** (i) Solve differential equation $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$ **03**

(ii) Solve differential equation $(D^2 + 1)y = \sin x$ **04**

(b) (i) Using variation of parameter solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$ **04**

(ii) Solve $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = e^{3x}$ **03**

Q.5 **(a)** (i) Form partial differential equation from **03**

(A) $z = f(x^2 + y^2)$ (B) $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

(ii) Solve $(mz - ny)p + (nx - lz)q = ly - mx$ **04**

(b) (i) Solve $p^2 + q^2 = x + y$ **03**

(ii) Solve (A) $z = px + qy + 2\sqrt{pq}$ (B) $p^2 + q^2 = 2$ **04**

OR

Q.5 **(a)** A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = \sin\frac{\pi x}{l}$ from which it is released at time t = 0. Show that the displacement of any point at distance x from one end at time t is given by $y(x,t) = a\sin\frac{\pi x}{l}\cos\frac{\pi ct}{l}$. **07**

(b) (i) Find the Z- transform of ka^k, $k \geq 0$ **03**

(ii) Solve deference equation $U_{k+1} + U_k = 1$ if $U_0 = 0$ **04**

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