Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC– SEMESTER I– • EXAMINATION –WINTER- 2016

Subject Code: X10001 **Date:10/01/2017**

Subject Name: Mathematics-I

Time: 10:30 AM to 1:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** For which value of k, the system of equations **07**

$2x+2y+5z=1, 3x+2y+5z=2k, 3x+4y+10z=k^2$

Have solution? Find the solution in each case.

(b) Find the minimum value of $x^2+y^2+z^2$ given that $xyz=8$. **07**

Q.2 **(a)** Solve $x\cos\left(\frac{x}{y}\right)\{ydx+xdy\}=y\sin\left(\frac{y}{x}\right)\{xdy-ydx\}$. **07**

(b) Trace the curve $x^3+y^3=3ax, a>0$. **07**

Q.3 **(a)** Integrate $f(x,y)=x^2+y^2$ over the triangular region with vertices $(0,0),(1,0),(0,1)$. **07**

(b) Evaluate $\oint_C [(x^2-\cosh y)dx+(y+\sin x)dy]$ by Green's theorem , where C is the rectangle with vertices $(0,0),(\pi,0),(\pi,1),(0,1)$. **07**

Q.4 **(a)** Find the orthogonal trajectories of the family of the circle $x^2+y^2=2ax$ in the polar form. **07**

(b) Find the rank of Matrix $A=\begin{bmatrix}1 & 2 & 3 & -1\\ -2 & -1 & -3 & -1\\ 1 & 0 & 1 & 1\\ 0 & 1 & 1 & -1\end{bmatrix}$ by row operation method and determinant method both. **07**

Q.5 **(a)** If $e^u=(x^3+y^3+z^3-3xyz)$, prove that $\frac{1}{9}\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2=\frac{-1}{(x+y+z)^2}$. **07**

(b) (1) Solve: $(x^2-4xy-2y^2)dx+(y^2-4xy-2x^2)dy=0$. **07**

(2) Solve : $\frac{dy}{dx}-y\cot x=2x\sin x$

Q.6 **(a)** Trace the curve : $r^2=a^2\cos 2\theta, a>0$. **07**

(b) Sketch the region of integration and evaluate by reversing the order of integration for $\int_0^{\frac{3}{2}}\int_3^{6-2x} xdydx$. **07**

Q.7 **(a)** Show that $\bar{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotation. **07**

(b) Find the Matrix A if $A^{-1} = \begin{bmatrix} -1 & -3 & -3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$ by Gauss- Jordan Method. **07**

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