Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC– SEMESTER II– • EXAMINATION –WINTER- 2016

Subject Code: X20001 **Date:31/12/2016**

Subject Name: Mathematics-II

Time: 02:30 PM to 5:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) Define Beta and Gamma function also evaluate $\int_0^\infty \frac{x^4(1+x^5)}{(1+x)^{15}}dx$ **07**

(b) (1) Prove that $B(m,n)=B(n,m)$. **03**

(2) Evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$. **04**

Q.2 (a) State Convolution theorem, find $L^{-1}\left[\frac{1}{S(S^2+4)}\right]$ **07**

(b) (1) Using partial fraction method find inverse Laplace Transform of **04**

$$\frac{(3S^2+2)}{(S+1)(S+2)(S+3)}$$

(2) State and prove First Shifting theorem for Laplace Transform. **03**

Q.3 (a) Find the Fourier Series to represent $f(x)=x^2,\ -\pi<x<\pi$. **07**

Also deduce $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}=\frac{\pi^2}{12}$

(b) Express e^{ax} as a cosine series in $0<x<\pi$. **07**

Q.4 (a) Find Fourier Integral of **07**

$$f(x) \begin{array}{ll} =1 & -1<x<1 \\ =0 & Otherwise \end{array}.$$

Hence evaluate $\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}d\lambda$.

(b) Find the Fourier Series to represent **07**

$$f(x) \begin{array}{ll} =-\pi\ , & -\pi<x<0 \\ =x\ , & 0<x<\pi \end{array}.$$

Also deduce $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+...$

Q.5 **(a)** (1) Solve $x\frac{dy}{dx}+y+1=0$. **03**

(2) Solve initial value problem **04**

$y''+y'-2y=0,\ y(0)=4, y'(0)=-5.$

(b) Find the general solution of $y''+9y=\sec 3x$ by method of variation parameters. **07**

Q.6 **(a)** Using Laplace Transform, solve the initial value problem **07**

$y'''+2y''-y'-2y=0,\ y(0)=1, y'(0)=2, y''(0)=2$ at $t=0$

(b) Find the solution of differential equation $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=3e^{-2x}$ by the method of Undetermined Co-efficient. **07**

Q.7 **(a)** State Convolution theorem, find z-transformation of **07**

$$f_1(k)=\{2,3,4\},\ f_2(k)=\{-1,2,3\}$$

(b) Solve by method of separation of variables, $u_{xx}=16u_y$ **07**

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