Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC - SEMESTER–II • EXAMINATION – WINTER 2013

Subject Code: X20001 Date: 18-12-2013

Subject Name: Mathematics-II

Time: 02.30 pm - 05.30 pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** 1) Define Beta function. Compute $\beta(2.5,1.5)$. **03**

2) Prove that $\int_0^1 x^3\left(1-\sqrt{x}\right)^5 dx = 2\beta(8,6)$. **04**

(b) 1) Form the partial differential equation from $z = ax + by + a^2 + b^2$ **03**

2) Solve $\frac{\partial^2 z}{\partial x^2} = xy$ **04**

Q.2 **(a)** Solve $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{-x} + \sin 2x$ **07**

(b) Solve $y'' + y = \tan x$ by the method of variation of parameter. **07**

OR

(b) Solve $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 8y = 65\cos(\log x)$. **07**

Q.3 **(a)** Find the Fourier series expansion of $f(x) = 2x - x^2$ in $(0,3)$, $f(x+3) = f(x)$. **07**

(b) State Convolution theorem and using it, evaluate $L^{-1}\left\{\frac{s}{(s+2)(s^2+9)}\right\}$. **07**

OR

Q.3 **(a)** Obtain a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$, $f(x+2\pi) = f(x)$. **07**

(b) Using Laplace transform method, Solve $y'' + y = t,\ y(0) = 1, y'(0) = -2$ **07**

Q.4 **(a)** Express $f(x) = x$ as a half range cosine series in $0 < x < 2$. $f(x+4) = f(x)$. **07**

(b) 1) Find $L\{(t+2)^2 e^t\}$. **03**

2) Find $L^{-1}\left\{\tan^{-1}\left(\frac{2}{s}\right)\right\}$ **04**

OR

Q.4 **(a)** Find a Fourier series to represent x^2 in the interval $(-l, l)$, $f(x+2l) = f(x)$. **07**

(b) 1) Find $L^{-1}\left\{\frac{1}{s^2 - 5s + 6}\right\}$. **03**

2) Find $L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\}$. **04**

Q.5 **(a)** 1) Solve $p + q = \sin x + \sin y$. **03**

2) Solve $x(y-z)p + y(z-x)q = z(x-y)$. **04**

(b) Express the function $f(x)=\begin{cases}1, & |x|<1\\ 0, & |x|>1\end{cases}$ as a Fourier integral. Hence evaluate $\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}\,d\lambda$. **07**

OR

Q.5 **(a)** Solve the equation $3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0,\quad u(x,0)=4e^{-x}$ by the method of separation of variables. **07**

(b) 1) Solve $(D^4-4D^2+4)y=0$ **03**

2) Solve $p(1+q)=qz$. **04**

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