

GUJARAT TECHNOLOGICAL UNIVERSITY**PDDC - SEMESTER-II • EXAMINATION – WINTER • 2014****Subject Code: X20001****Date: 23-12-2014****Subject Name: Mathematics - II****Time: 02:30 pm - 05:30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) (1) Define Gamma and Beta Functions. Prove that $B(m, n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx$. **05**

(2) Find $L(e^{-t}t^3)$. **02**

(b) Express $f(x) = x \cos x$ as a Fourier Series in $(-\pi, \pi)$. **07**

Q.2 (a) Using Laplace transforms solve the initial value problem $y'' + y = \sin 2t$,
where $y(0) = 0$ & $y'(0) = 0$. **07**

(b) Solve partial differential equation $pz - qz = z^2 + (x + y)^2$. **07**

OR

(b) Solve: $x^2y'' - 4xy' + 6y = 21x^{-4}$. **07**

Q.3 (a) (1) Find the Laplace transform of $t \sin t$ **03**

(2) Find the Laplace transform of $e^{-t}(t^2 - 2t + 4) \sin t$ **04**

(b) Using the method of variation of parameters find the general solution of the differential equation $(D^2 + 2D + 1)y = 3x^{3/2}e^x$. **07**

OR

Q.3 (a) State the Convolution theorem for Laplace inverse transform . **07**

Using it find the Laplace Inverse transform of $\frac{s}{(s^2 + a^2)^2}$.

(b) Solve the equation $u_x = 2u_t + u$ given $u(x, 0) = 4e^{-3x}$ by method of separation of variables. **07**

Q.4 (a) Show that $x^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}$ in the interval $-\pi \leq x \leq \pi$. Hence **07**

deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$.

(b) Using Beta-Gamma functions to prove that $\left(\int_0^{\infty} \sqrt{x} e^{-x^2} dx \right) \times \left(\int_0^{\infty} \frac{e^{-x^2}}{\sqrt{x}} dx \right) = \frac{\pi}{2\sqrt{2}}$. **07**

OR

Q.4 (a) Find the Fourier cosine transform of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$. **07**

(b) Solve $(D^2 + 2D + 10)y + 37 \sin 2x = 0$. **07**

Q.5 (a) (1) Eliminate the arbitrary function from the equation $z = xy + f(x^2 + y^2)$. **05**

(2) Solve : $\frac{\partial^2 z}{\partial x^2} = \sin x$ **02**

(b) (1) Define Z-transform. Find the Z-transform of the sequence $\{a^m\}, m \geq 0$. **05**

(2) Prove that $L(1) = \frac{1}{s}$. **02**

OR

Q.5 (a) (1) Eliminate the arbitrary function from the equation $f(x+y+z, x^2+y^2+z^2) = 0$. **05**

(2) Solve: $\frac{\partial^2 z}{\partial x \partial y} = x^3 + y^3$. **02**

(b) (1) State the linearity property of Z-transform. Find the Z-transform of **05**

$$f(k), \text{ where } f(k) = \begin{cases} 7^k, & k < 0 \\ 5^k, & k \geq 0. \end{cases}$$

(2) State the relation between Beta-Gamma functions. **02**
