**Roll No.** **Enrollment No.**

# C U SHAH COLLEGE OF ENGINEERING & TECHNOLOGY, WADHWAN-CITY

## B. E. all Semester-II Mid Semester Examination March 2009

## Subject: Mathematics-II

**Date: 23/02/2009** **Time: 03:00p.m. to 05:00p.m.** **Max. Marks: 40**

**Instructions:** 1) Attempt all questions.

2) Make suitable assumptions whenever necessary.

3) Figures to the right indicate full marks.

**Q. 1.** **(a)** State Cauchy-Schwarz inequality. Verify Triangular inequality for $\boldsymbol{u} = (1, -2,\ 3)$ and $\boldsymbol{v} = (2,\ 0,\ 1)$. **[5]**

**(b)** Attempt any two. **[10]**

**(I)** Determine which of the following are subspaces of $R^3$.

(i) all vectors of the form $(a\ ,1,\ 1)$.

(ii) all vectors of the form $(a,\ b,\ c)$, where $b = a + c$.

**(II)** Show that $\mathrm{F}[0,\ 1]$ the set of all real valued functions on $[0,\ 1]$ is a real vector space.

**(III)** Find inverse of $A = \begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$ by Gauss-Jordan Method.

**(IV)** Solve the following system of linear equations by Gauss-Jordan Method.

$$\begin{aligned} x + y\ \ + 2z &= 9 \\ 2x + 4y - 3z &= 1 \\ 3x + 6y - 5z &= 0 \end{aligned}$$

**(V)** State Pythagorean Theorem and verify it for $\boldsymbol{u} = (1, -3,\ 2)$ and $\boldsymbol{v} = (4,\ 2,\ 1)$.

**Q. 2.** **(a)** Show that $R^n$ is a real vector space with the standard operations. **[5]**

**(b)** Attempt any two. **[10]**

**(I)** Check whether the vectors $(1,\ 1,\ 1)$, $(1,\ 1,\ 0)$ and $(1,\ 0,\ 0)$ are Linear Independent or not.

**(II)** Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 1 & 1 \end{bmatrix}$.

**(III)** Find reduced row-echelon form of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

**(IV)** Find the vector component of $\boldsymbol{u}$ along $\boldsymbol{a}$ and the vector component of $\boldsymbol{u}$ orthogonal to $\boldsymbol{a}$ for $\boldsymbol{u} = (3,\ 0,\ 4),\ \boldsymbol{a} = (2,\ 3,\ 3)$.

**(V)** Show that $\mathrm{T}: \mathrm{R}^2 \to \mathrm{R}^2$ defined by $\mathrm{T}(x,\ y) = (x - y,\ x + y)$ is a Linear Transformation.

**Q. 3.** Attempt any two. **[10]**

**(a)** Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$.

**(b)** Determine which of the following are subspaces of $M_{22}$.

(i) all 2 × 2 matrices with integer entries.

(ii) all 2 × 2 matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with $a + b + c + d = 0$.

**(c)** Find $\mathrm{T}^{-1}$ for the Linear Transformation $\mathrm{T}: \mathrm{R}^3 \to \mathrm{R}^2$ defined by $\mathrm{T}(x,\ y,\ z) = (x - y,\ \mathrm{z})$.

**(d)** (i) Find the distance D between the point $(2, -1,\ 2)$ and the plane $3x - y + 2z = 1$.

(ii) Find the volume of the parallelepiped with sides $\boldsymbol{u} = (2, -6,\ 2), \boldsymbol{v} = (0,\ 4, -2)$ and $\boldsymbol{w} = (2,\ 2, -4)$.

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